**Purpose**

When inductors, capacitors and resistors are placed in combination in series as shown at right, a phenomena known as resonance occurs. This laboratory will determine the resonant frequency of an LCR circuit and use it to calculate the inductance of the inductor.

**Theory**

In LCR circuits, the impedance to the current is caused by the reactance of the inductor and capacitor, and by the resistance of the resistor. Unlike in pure resistive circuits, the reactances do not simply add in series. Rather, because the voltages of each element of the circuit are out of phase, one must use phasor diagrams to determine the net impedance. The voltages through the capacitor and the inductor are out of phase by 180 degrees and thus subtract giving a vector

1) \[ V_{LC} = V_L - V_C \]

The root-mean-square voltage, \( V_{RMS} \), can be found using the Pythagorean theorem.

2) \[ V_{RMS} = \sqrt{(V_L - V_C)^2 + V_R^2} \]

Using Ohm's Law, we get:

3) \[ I_{RMS} Z = \sqrt{(I_{RMS} X_L - I_{RMS} X_C)^2 + (I_{RMS} R)^2} \]

With some factoring, this expression becomes:

4) \[ I_{RMS} Z = \sqrt{(I_{RMS})^2[(X_L - X_C)^2 + (R)^2]} \]

or

5) \[ I_{RMS} Z = I_{RMS} \sqrt{(X_L - X_C)^2 + (R)^2} \]
Finally, it can be seen that the impedance for such a circuit is:

\[ Z = \sqrt{(X_L - X_C)^2 + R^2} \]  

When the capacitive reactance is equal to the inductive reactance, the impedance reaches its minimum value: \( Z = R \). At this frequency, the current reaches its maximum value. At low frequencies, the capacitive reactance dominates, while at high frequencies, the inductive reactance dominates.

The capacitive reactance is given by:

\[ X_C = \frac{1}{\omega C} \]  

The inductive reactance is given by:

\[ X_L = \omega L \]  

If \( X_C = X_L \), we find that:

\[ \omega L = \frac{1}{\omega C} \]  

or

\[ \omega^2 = \frac{1}{LC} \]  

Because \( \omega = 2\pi f \), this last equation can be used to find the resonant frequency in terms of L and C:

\[ f = \frac{1}{2\pi \sqrt{LC}} \]  

**Procedure**

Using the circuit board, wire a copper coil (inductor) in series with the 3.3 MΩ resistor and a 1 to 10 \( \mu \)F capacitor (See diagram below). Power your circuit using the Pasco Power Supply II. Be certain that it is plugged in as shown and powered on along with the Pasco 750. Turn on your computer and start up Data Studio. Choose the Power Amplifier sensor.
Unlike in the past, we will not be using Data Studio to collect data. Rather, we will use it to drive the power amplifier, and use the multimeter to collect data. Set the waveform to sine wave, the voltage to 5V, and the frequency to 100 Hz. Turn the power supply to on, and begin collecting data by viewing the multimeter. Be certain that it is connected as shown below. If your currents are below 300 mA, use the lower left hand input. Otherwise, use the upper left hand input to read current.

Step up the frequency of the sine wave by 100 Hz until you reach 1500 Hz (you should have encountered resonance by then). To nail down the shape of the curve near the resonance, starting at 100 Hz below your highest current reading, record the current and frequency in 10 Hz increments to 100 Hz higher than your previously highest current reading. As usual, record uncertainties with all of your data.

Next, measure the dimensions of your coil. Estimate the cross-sectional area and length as well as the number of turns. Use the equation

$$L=\mu_0 a N^2 \left(0.5 + \frac{S_1}{12}\right) \ln \left(\frac{S}{S_1}\right) - 0.84834 + 0.2041 S_1$$  \hspace{1cm} (12)$$

to estimate the inductance of the coil. Here, $a$ is the mean radius of the coil, $b$ is the width, $c$ is the thickness, $S_1$ is given by $(c/2a)^2$, and $N$ is the total number of turns of the solenoid. All measurements are in meters. $L$ is in micro-Henrys. This is known as the Brooks equation for an air coil.
Analysis

Create a graph of your current versus your frequency data. Estimate the resonant frequency from the graph with uncertainty. Based on this and equation 11) estimate the inductance of the coil. Be certain to estimate the uncertainty in the inductance based on the uncertainties in the capacitance and the frequency.

Next, using the Brooks equation, estimate the inductance of the coil based on its geometric properties. Estimate the uncertainties in this inductance based on the uncertainties of the physical dimensions.

Are the inductances calculated using different methods equal within experimental uncertainty? If these two inductances do not match, what factors could contribute? Be specific in your answer. Why is the Brooks equation used instead of the usual inductance equation from your book? Are there any anomalies in your data? If so, explain them.

References

http://www.ee.surrey.ac.uk/Workshop/advice/coils/air_coils.html