DC Circuit Analysis using Kirchoff's Rules

Object: The purpose of this lab will be to test Kirchoff’s Rules and to use them in the analysis of some basic DC circuits.

Reference: Resnick & Halliday (Chapter 27, section 4-7)

Apparatus:
1. Resistors (27Ω, 47Ω, 100Ω)
2. D-cell Batteries (1.5 V)
3. Digital Multimeter

Background:
Circuits in which the resistors are not in simple series or parallel groupings, or in which there are batteries in parallel paths, can be analyzed by means of Kirchoff’s Rules. Kirchoff’s two laws are:

I. The sum of all currents arriving at any branch point in a circuit is equal to the sum of the currents leaving that junction.
II. Around any closed loop the sum of the emfs is equal to the sum of the iR drops.

When applying Kirchoff’s rules keep in mind the following:

I. A branch point is any point in a circuit where three or more wires are joined.
II. A branch is a segment of a circuit containing resistors and/or emfs between two branch points.
III. A loop is any closed loop in the circuit containing resistors and/or emfs.

Procedure:
1. Assign a direction and a symbol to the current in each branch of the circuit.

2. Apply Kirchoff’s first rule at the branch points. If there are \( n \) branch points, apply the rule \((n-1)\) times.

3. Choose loops that will be used and assign a direction to be followed in going around the loop. Use enough loop so that each current is contained in at least one loop equation.

4. Write the loop equations keeping in mind that as you go around the loop:
   a. emfs are positive if passed through from negative to positive terminal. Negative if passed through from positive to negative terminal.
   b. \( iR \) drops are positive (remember: a positive drop decreases the potential!!) if you pass through the resistor in the direction of the current flow; negative if opposite the direction of the current flow.
The following examples will help clarify the procedure.

**Example 1:** Find the currents in each branch of the circuit below.

1. Assign variables with subscripts to currents in each branch and indicate their directions with arrows:
   A.
   B.

The directions you choose are arbitrary, usually chosen with an educated guess. If you picked the wrong direction, a (-) sign will come out in the algebra. The important thing here is to be CONSISTENT with the directions you have chosen and the currents in each part of the circuit, as indicated in diagram B of the above step. (i.e.- The middle branch has a current, called “i2” everywhere along it and always pointing in the same direction.)

The important things are:
   a. Assign a symbol (such as $i_1$ or $i_2$) to the current in each branch.
   b. Assign a direction to the current in each branch. Do not spend much time trying to decide which way the current is "really" going, but be sure you are consistent (i.e.- The middle branch has a current, called “i2” everywhere along it and always pointing in the same direction). If your choice is opposite the actual current flow, the current will come out of the algebra to be negative.
2. Using Rule I and Diagram A we get an algebraic equation relating the three currents in this circuit:

\[ i_1 = i_2 + i_3 \]  
(Eq. 1)

3. Choose loops and directions for loops.
   Here are three choices:

Note: Any of these is correct. Also, the directions shown for the loops (all clockwise in the above examples) could just as well be counter clockwise. Again, labeling your diagram well so you can always look at it and tell which choices for directions that you made, and so that you can obtain correct algebraic equations in applying Kirchoff’s Rules is the most important thing in assigning directions.
4. Writing the loop equations for the first arrangement in (3):

Upper Loop:

\[-\varepsilon_1 + \varepsilon_2 = i_1 R_1 + i_2 R_2\]  
(Eq. 2)

Lower Loop:

\[-\varepsilon_2 = -i_2 R_2 + i_3 R_3\]  
(Eq. 3)

Example 2:

In the circuit shown, determine the current in each branch of the circuit.

at point a, applying Rule I:  
\[i_1 + i_3 = i_2\]

From Loop I:  
\[-2 - 3 = -(1)i_1 - (5)i_2\]

From Loop II:  
\[+3 = (5)i_2 + (4)i_3\]

\[i_1 + i_3 = i_2\]

\[-5 = -i_1 - 5i_2\]

Collect Equations:  
\[3 = 5i_2 + 4i_3\]

We have three equations and three unknowns. Solve simultaneously.

\[i_1 = 1.03A; \quad i_2 = 0.793A; \quad i_3 = -0.241A\]

The negative sign indicates that \(i_3\) is opposite the direction originally chosen.
Pre-Lab Assignment PROBLEM 1)

Rule I at branch point:

Rule II for top loop:

Rule II for bottom loop:

Solve for currents:
Experiment:

a) Wire the circuit shown in the Pre-Lab assignment, Problem 1, using 1.5 volt batteries and the resistors as close as possible to the values shown. Use a voltmeter to measure the emf of the batteries. If it differs from 1.5 volts by more than ± .1 volt get another battery. Re-solve problem 1 using those actual measured values of battery emfs and resistance. Get values for the three currents, algebraically.

b) When your circuit is set up (have your instructor check it before making the final connections to the batteries) measure the currents \( i_1 \), \( i_2 \), and \( i_3 \), by inserting a milli-ammeter in each of the branches corresponding to these currents. (When using your multimeter, be sure to change the probes and dial to the appropriate settings for an ammeter!)

Compare the values obtained with your calculated values. (Find % differences)

PROBLEM 2)

a. Repeat Problem 1 for the following circuit. Draw your own currents and loop directions.

b. Wire the circuit and find the currents experimentally.

Report: Next week you need to turn in the following:

1. A neat sketch of circuit #1 (from Problem #1), labelled with the actual values of Emf’s and R’s you used. Be sure to indicate the assumed current directions and the “loops” you used.

2. The equations you got from applying Kirchoff’s laws to this circuit (they will be almost like the equations at the bottom of page 4).

3. A detailed page of the algebra done to solve for the three unknown currents for circuit #1.

4. A table listing predicted (algebraically determined) values for the currents, experimentally determined currents, and % errors in your predictions.

5. A neat sketch of circuit #2 (from Problem #2), labelled up real good...

6. The three equations you got from applying Kirchoff’s laws to this circuit.

7. A table listing the theoretical and experimental values of the currents and the % errors for circuit #2.

8. All the above packaged and presented together neatly. Maybe a nice picture on the front... I don’t know, you make the call.